Wave Axis Slope

Wave transport of both heat and momentum in the north-south direction are very important processes that affect wave intensity. Start with the zonal component of the equation of motion in flux form, and then apply the perturbation method.

Flux Form of Zonal Momentum Equation

To derive the flux form of the east-west component of the large-scale equation of motion, using pressure as the vertical co-ordinate, start with

$$\frac{du}{dt} = -\frac{\partial \phi}{\partial x} + fv.$$

Expand the total derivative,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \omega\frac{\partial u}{\partial p} = -\frac{\partial \phi}{\partial x} + fv. \qquad (1)$$

Multiply the equation of continuity by u,

$$u\frac{\partial u}{\partial x} + u\frac{\partial v}{\partial y} + u\frac{\partial \omega}{\partial p} = 0. \qquad (2)$$

Add equations (1) and (2), (using the product rule),

$$\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial u\omega}{\partial p} = -\frac{\partial \phi}{\partial x} + fv. \qquad (3)$$

The second, third, and fourth terms now involve the flux of zonal momentum, u, along each of the three co-ordinate axes. For example, uv is the flux of u in the north-south direction, and $u\omega$ is the flux of u in the vertical direction. Also, each term is a **flux divergence**, because it is a change of the flux in the same direction as the flux.

The Perturbation Method

Equation (3) governs changes to the zonal momentum by all scales of motion. To explicitly show two of these circulations, waves and the jet stream, use the perturbation method.

Assume that the **mean flow** consists of three meridional circulations and jet streams, and that the **perturbation flow** consists of the mid-latitude waves. Also, assume that the mean vertical motion, $\overline{\omega}$ is negligible compared to the wave vertical motion, ω'.

$$u = \overline{u} + u'$$

$$v = \overline{v} + v'$$

$$\omega = \omega'$$

Substitute into equation (3),

$$\frac{\partial\left(\overline{u}+u'\right)}{\partial t} + \frac{\partial\left(\overline{u}+u'\right)\left(\overline{u}+u'\right)}{\partial x} + \frac{\partial\left(\overline{u}+u'\right)\left(\overline{v}+v'\right)}{\partial y} + \frac{\partial\left(\overline{u}+u'\right)\omega'}{\partial p} = -\frac{\partial \phi}{\partial x} + f\left(\overline{v}+v'\right) .$$

Or,

$$\frac{\partial\left(\overline{u}+u'\right)}{\partial t} + \frac{\partial\left(\overline{u}\,\overline{u}+2\overline{u}u'+u'u'\right)}{\partial x} + \frac{\partial\left(\overline{u}\,\overline{v}+\overline{u}v'+u'\overline{v}+u'v'\right)}{\partial y} + \frac{\partial\left(\overline{u}\omega'+u'\omega'\right)}{\partial p} = -\frac{\partial \phi}{\partial x} + f\left(\overline{v}+v'\right)$$

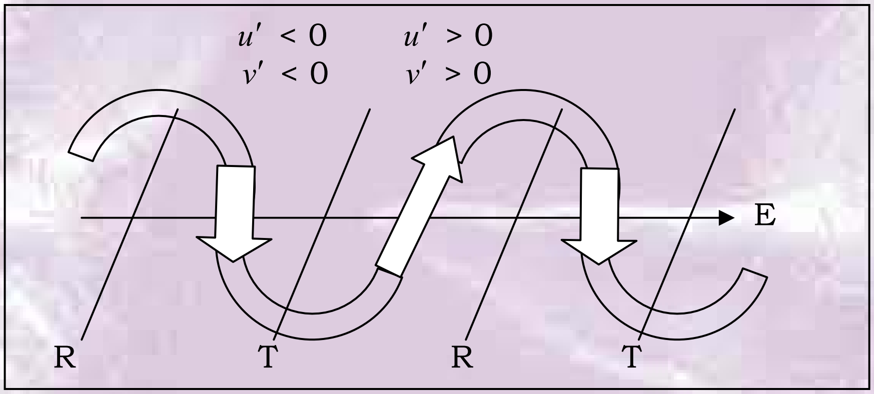
Next, average this equation around a latitude circle (in an easterly direction). Note that means of perturbations are zero, $\overline{u'} = 0$. Also, $\overline{\overline{u}u'} = \overline{u}\,\overline{u'} = 0$, because a mean quantity is similar to a constant. Then,

$$\boxed{\frac{\partial \overline{u}}{\partial t} + \frac{\partial\left(\overline{u}\,\overline{u}+\overline{u'u'}\right)}{\partial x} + \frac{\partial\left(\overline{u}\,\overline{v}+\overline{u'v'}\right)}{\partial y} + \frac{\partial\left(\overline{u'\omega'}\right)}{\partial p} = f\overline{v}} \quad . \qquad (4)$$

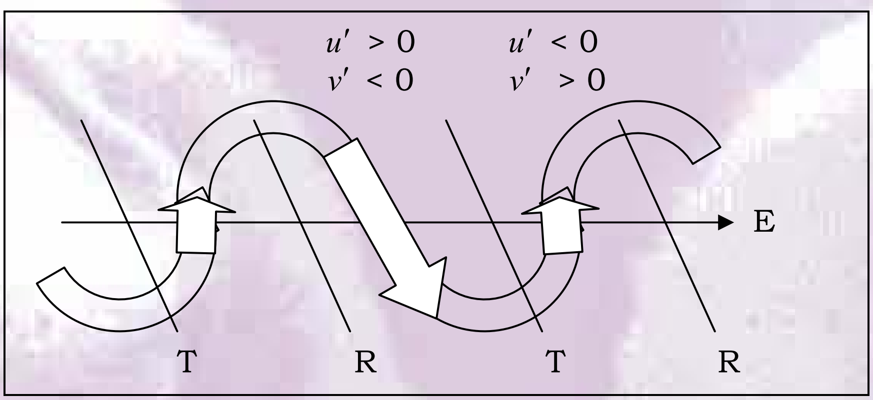
Equation (4) shows the processes that can change the mean zonal wind, $\overline{u}$. Three processes involve the mean circulation (terms with mean quantities), and three involve the waves (terms with perturbation quantities). Remember, the three wave processes explicitly appear only because the perturbation method was used.

Wave Transport of Zonal Momentum

The north-south flux of zonal momentum by the waves, $\overline{u'v'}$, is especially important. It turns out that this flux is toward the pole when the wave axis is positively tilted, and toward the equator when the wave axis is negatively tilted. Consider a wave that is positively tilted, as below. East of the trough axis v is from the south (positive); west of the trough axis v is from the north (negative). Thus, v' is positive east of the trough axis, while v' is negative west of the trough axis. East of the trough axis u is slightly positive, while west of the trough axis u is slightly negative. Thus, u' is positive east of the trough axis, and u' is negative west of the trough axis. (Remember, $u' = u - \bar{u}$.) The product of u' and v' is positive everywhere, so, $\overline{u'v'}$ is positive, and the waves transport zonal momentum northward. As a result, energy is flowing out of the waves and into the jet, and the waves should weaken while the jet should strengthen.



Consider a wave that is negatively tilted, as below. Again, v' is positive east of the trough axis, while v' is negative west of the trough axis. However, the sign of u' is reversed from the previous example; u' is negative east of the trough axis, and u' is positive west of the trough axis. The product of u' and v' is



negative everywhere. So, $\overline{u'v'}$ is negative; the waves transport zonal momentum southward. As a result energy is flowing out of the jet and into the waves; the jet should weaken while the waves should strengthen.

Wave Transport of Heat

Waves almost always transport heat toward the pole, because the warmest air is almost always on the east side of the trough axis. Thus, T' is positive on the east side of the trough axis, as is v'. On the west side of the trough axis, T' is negative as is v'. So, the poleward heat flux, $\overline{v'T'}$, is positive.