Vector Derivation of Geostrophic Vorticity

The vorticity of the geostrophic wind is found by performing a

$$\vec{k}\cdot\nabla_p\times \quad \text{on} \quad f_o\vec{V}_g = \vec{k}\times\nabla_p\phi.$$

$$\vec{k}\cdot\nabla_p\times\left(f_o\vec{V}_g\right) = \vec{k}\cdot\nabla_p\times\left(\vec{k}\times\nabla_p\phi\right).$$

Noting that

$$\zeta_g = \vec{k}\cdot\nabla\times\vec{V}_g,$$

$$f_o\zeta_g = \vec{k}\cdot\nabla_p\times\left(\vec{k}\times\nabla_p\phi\right).$$

Earlier we found that

$$\vec{k}\times\nabla_p\phi = \vec{k}\times\left(\vec{i}\frac{\partial\phi}{\partial x} + \vec{j}\frac{\partial\phi}{\partial y}\right) = \left(\vec{j}\frac{\partial\phi}{\partial x} - \vec{i}\frac{\partial\phi}{\partial y}\right),$$

so

$$\nabla_p\times\left(\vec{k}\times\nabla_p\phi\right) = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y}\right)\times\left(\vec{j}\frac{\partial\phi}{\partial x} - \vec{i}\frac{\partial\phi}{\partial y}\right) =$$

$$\vec{i}\times\vec{j}\frac{\partial^2\phi}{\partial^2 x} - \vec{j}\times\vec{i}\frac{\partial^2\phi}{\partial^2 y} = \vec{k}\left(\frac{\partial^2\phi}{\partial^2 x} + \frac{\partial^2\phi}{\partial^2 y}\right) = \vec{k}\,\nabla_p^2\,\phi$$

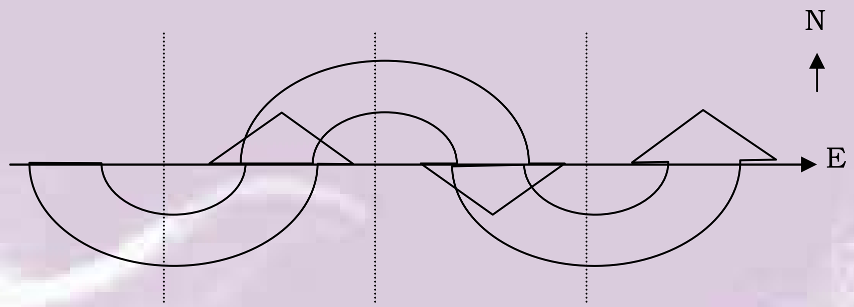
Finally,

$$f_o\zeta_g = \vec{k}\cdot\vec{k}\nabla_p^2\,\phi,$$

and,

$$\boxed{f_o\zeta_g = \nabla_p^2\,\phi}\ .$$

The geostrophic vorticity is determined by just the geopotential. And the curvature of the geopotential determines whether the vorticity is positive or negative. As one would expect, the maximum positive geostrophic vorticity occurs were the curvature is most positive, along the trough axis. Similarly, the maximum negative geostrophic vorticity occurs were the curvature is most negative, along the ridge axis.



	T		R		T	
slope:		+		-		+
curvature:	+		-		+	
vorticity:	+		-		+	