Surface Layer

The portion of the atmosphere where friction is important is called the boundary layer, or the **planetary boundary layer**, PBL. It is usually divided into two layers where each has different characteristics. The lower layer is called the **surface layer**, and is approximately 10 m thick.

To determine how the wind varies with height in the surface layer, start with the mixing length formulation of the stress corresponding to the vertical eddy flux of zonal momentum,

$$\tau_{zx} = \rho\, l_x^2 \left| \frac{\partial\, \bar{u}}{\partial\, z} \right| \frac{\partial\, \bar{u}}{\partial\, z} .$$

Rearrange,

$$\left(\frac{\tau_{zx}}{\rho} \right) \frac{1}{l_x^2} = \left| \frac{\partial\, \bar{u}}{\partial\, z} \right| \frac{\partial\, \bar{u}}{\partial\, z} .$$

In the surface layer, the stress and the density are assumed to be constant with height. So, a new quantity, the **friction velocity,** u_***,** is defined as,

$$u_* = \sqrt{\left| \frac{\tau_{zx}}{\rho} \right|} . \qquad (1)$$

Thus,

$$\frac{u_*^2}{l_x^2} = \left(\frac{\partial\, \bar{u}}{\partial\, z} \right)^2 .$$

Solving for the vertical wind shear,

$$\left| \frac{\partial\, \bar{u}}{\partial\, z} \right| = \frac{u_*}{|\, l_x|} .$$

In the surface layer, the mixing length is assumed to be proportional to the height above ground, $|\, l_x| = k\, z$, where the proportionality factor, k, is the **von Karman constant** (~0.38),

$$\left| \frac{\partial\, \bar{u}}{\partial\, z} \right| = \frac{u_*}{k\, z} .$$

In order to solve for the mean wind as a function of height, this equation needs to be integrated. First, multiply both sides by dz,

$$\left|\frac{\partial\,\bar{u}}{\partial\,z}\right|\,dz\;=\;\frac{u_*}{k\,z}\;dz\,.$$

Then assume the environmental zonal wind, $\bar{u}$, changes more with height, z, than any other independent variable,

$$d\,\bar{u}\;=\;\left|\frac{\partial\,\bar{u}}{\partial\,z}\right|\,d\,z\;=\;\frac{u_*}{k\,z}\;d\,z\;=\;\frac{u_*}{k}\;d\ln z\,.$$

Now integrate from the lower boundary, where $z = z_o$ and $\bar{u}=0$, to the arbitrary height z, where $\bar{u}=\bar{u}$ (z). The result is the **logarithmic wind profile**,

$$\boxed{|\bar{u}|=\frac{u_*}{k}\ln\frac{z}{z_o}}\quad. \tag{2}$$

The **roughness parameter**, z_o, depends on the characteristics of the surface, and is approximately 1/30th the height of surface objects.

The friction velocity can be determined from wind observations at the **anemometer level**, z_A. At this level,

$$|\bar{u}_A|\;=\;\frac{u_*}{k}\;\ln\frac{z_A}{z_o}\,.$$

Solve for the friction velocity,

$$u_*\;=\;\frac{k\;|\bar{u}_A|}{\ln\dfrac{z_A}{z_o}}\,. \tag{3}$$

Then, the stress can be determined from only large-scale variables. From equation (1),

$$|\tau_{zx}|\;=\;\rho u_*^2\;=\;\rho\left(\frac{k\,\bar{u}_A}{\ln\dfrac{z_A}{z_o}}\right)^2.$$

And since,

$$\rho \overline{u'w'} = - \tau_{zx} .$$

the vertical eddy flux of zonal momentum in the surface layer can be determined.

The frictional force in the surface layer can be obtained by substituting equation (2) into

$$\frac{F_x}{M} = \nu \frac{\partial^2 u}{\partial z^2} ,$$

and substituting for u_* from equation (3),

$$\frac{F_x}{M} = - \frac{\nu \; u_A}{z^2 \; \ln \dfrac{z_A}{z_o}} .$$