Surface Fluxes of Heat and Moisture

To parameterize the transport of heat across the Earth's surface, assume the **vertical eddy heat flux**, $\rho\, C_p\, \overline{T'w'}$, is proportional to the vertical heat gradient, $\dfrac{\partial\, C_p\, \overline{T}}{\partial\, z}$, which is similar to what was assumed with momentum.

$$\rho\, C_p\, \overline{T'w'} \;=\; -\,\mu_{ex}\, \frac{\partial\, C_p\, \overline{T}}{\partial\, z}\,. \qquad (1)$$

The eddy coefficient of viscosity can be determined from previous equations,

$$\mu_{ex} \;=\; \rho\, l_x^2 \left|\frac{\partial\, \overline{u}}{\partial\, z}\right| \text{ and } \left|\frac{\partial\, \overline{u}}{\partial\, z}\right| \;=\; \frac{u_*}{|\, l_x|}\,.$$

Or,

$$\mu_{ex} \;=\; \rho\, l_x\, u_* \;=\; \rho\, k\, u_*\, z\,.$$

Since,

$$u_* \;=\; \frac{k\, |\overline{u}_A|}{\ln(z_A / z_o)}\,,$$

$$\mu_{ex} \;=\; \frac{\rho\, k^2\, |\overline{u}_A|\, z}{\ln(z_A / z_o)}\,.$$

Substitute into equation (1),

$$\rho\, C_p\, \overline{T'w'} \;=\; -\,\frac{\rho\, k^2\, |\overline{u}_A|\, z}{\ln(z_A / z_o)}\, \frac{\partial\, C_p\, \overline{T}}{\partial\, z} \;=\; -\,\frac{\rho\, k^2\, |\overline{u}_A|}{\ln(z_A / z_o)}\, \frac{\partial\, C_p \overline{T}}{\partial\, \ln z}\,.$$

Next, the vertical heat gradient is written using finite differences between the **observation level**, z_b, and the roughness height, z_o,

$$\frac{\partial\, C_p\, \overline{T}}{\partial\, \ln z} \;=\; \frac{C_p\, \left(\overline{T}_b - \overline{T}_o\right)}{(\ln z_b - \ln z_o)} \;=\; \frac{C_p\, \left(\overline{T}_b - \overline{T}_o\right)}{\ln(z_b / z_o)}\,.$$

The temperature at the roughness height is interpreted to be the temperature of the surface, $\overline{T}_s$,

$$\frac{\partial\, C_p\, \overline{T}}{\partial\, \ln z} = \frac{C_p\left(\overline{T}_b - \overline{T}_s\right)}{\ln\left(z_b / z_o\right)}.$$

And the vertical eddy heat flux becomes,

$$\rho\, C_p\, \overline{T'w'} = -\frac{\rho\, k^2\, |\overline{u}_A|}{\ln\left(z_A / z_o\right)}\, \frac{C_p\left(\overline{T}_b - \overline{T}_s\right)}{\ln\left(z_b / z_o\right)}.$$

Defining a **drag coefficient for heat**,

$$C_H = \frac{k^2}{\ln\left(z_A / z_o\right)\, \ln\left(z_b / z_o\right)},$$

$$\boxed{\rho\, C_p\, \overline{T'w'} = \rho\, C_p\, C_H\, |\overline{u}_A|\, \left(\overline{T}_s - \overline{T}_b\right)}\ .$$

A similar relationship for water vapor can be derived using a conservative quantity such as mixing ratio or specific humidity instead of sensible heat, $C_p\, T$.

The change of the large-scale heat with time due to just the eddy heat flux is given by,

$$\frac{\partial}{\partial\, t}\left(\rho\, c_p\, \overline{T}\right) = -\frac{\partial}{\partial\, z}\left(\rho\, c_p\, \overline{T'w'}\right).$$

Thus, it is important to know the eddy heat flux at the bottom of the boundary layer and at the top. While the value at the bottom is related to temperature differences between the ground and the air, the value at the top is due to other processes, such as cloud and turbulent fluxes.