The Quasi-geostrophic Governing Equations

The governing equations are the thermodynamic energy and vorticity equations,

$$\frac{\partial}{\partial t}\left(-\frac{\partial \phi}{\partial p}\right) = -\vec{V}_g \cdot \nabla_p \left(-\frac{\partial \phi}{\partial p}\right) + \omega \sigma$$

$$\frac{\partial}{\partial t}\,(f_o^{-1}\,\nabla_p{}^2\phi) = -\vec{V}_g \cdot \nabla_p\,(f_o^{-1}\,\nabla_p{}^2\phi + f) + f_o\,\frac{\partial \omega}{\partial p}.$$

Remembering that

$$\vec{V}_g = f_o^{-1}\,\vec{k}\times\nabla_p\,\phi,$$

one realizes that this is a set of two equations with two unknowns, ϕ and ω. Thus, the set can be solved for each unknown.

The Quasi-geostrophic Geopotential Tendency Equation

To form one equation for ϕ, operate on the energy equation with $-\frac{f_o}{\sigma}\frac{\partial}{\partial\, p}$ and add to the vorticity equation. First,

$$\left(-\frac{f_o}{\sigma}\frac{\partial}{\partial\, p}\right)\frac{\partial}{\partial t}\left(-\frac{\partial \phi}{\partial p}\right) =$$

$$+\left(-\frac{f_o}{\sigma}\frac{\partial}{\partial\, p}\right)\left(-\vec{V}_g \cdot \nabla_p\right)\left(-\frac{\partial \phi}{\partial p}\right) + \left(-\frac{f_o}{\sigma}\frac{\partial}{\partial\, p}\right)\omega\sigma \,.$$

Or, assuming a constant stability,

$$\frac{\partial}{\partial t}\left(\frac{f_o}{\sigma}\frac{\partial^2 \phi}{\partial\, p^2}\right) = -\left(\frac{f_o}{\sigma}\frac{\partial}{\partial\, p}\right)\left(-\vec{V}_g \cdot \nabla_p\right)\left(-\frac{\partial \phi}{\partial p}\right) - f_o\,\frac{\partial \omega}{\partial\, p}.$$

Add this to the vorticity equation,

$$\frac{\partial}{\partial t}\,(f_o^{-1}\,\nabla_p{}^2\phi) = -\vec{V}_g \cdot \nabla_p\,(f_o^{-1}\,\nabla_p{}^2\phi + f) + f_o\,\frac{\partial \omega}{\partial\, p}.$$

The result is,

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$$\frac{\partial}{\partial t}\left(f_o^{-1}\nabla_p^{\,2}\phi + \frac{f_o}{\sigma}\frac{\partial^2\phi}{\partial p^2}\right) =$$

$$-\vec{V}_g\cdot\nabla_p\left(f_o^{-1}\nabla_p^{\,2}\phi + f\right) - \left(\frac{f_o}{\sigma}\frac{\partial}{\partial p}\right)\left(-\vec{V}_g\cdot\nabla_p\right)\left(-\frac{\partial\phi}{\partial p}\right).$$

Or,

$$\boxed{\left(f_o^{-1}\nabla_p^{\,2} + \frac{f_o}{\sigma}\frac{\partial^2}{\partial p^2}\right)\frac{\partial\phi}{\partial t} = -\vec{V}_g\cdot\nabla_p\left(f_o^{-1}\nabla_p^{\,2}\phi + f\right) - \left(\frac{f_o}{\sigma}\frac{\partial}{\partial p}\right)\left(-\vec{V}_g\cdot\nabla_p\right)\left(-\frac{\partial\phi}{\partial p}\right)}.$$

Labeling each term: A = B + C

Term A is the three dimensional Laplacian of the geopotential tendency. The coefficients of the horizontal and vertical parts weight each part appropriately. Earlier, the second derivative of a sinusoidal quantity was shown to be proportional to the negative of that quantity. Therefore, *term A* is proportional to the negative of the geopotential tendency.

Term B, which was also discussed earlier, is the horizontal advection of absolute vorticity.

Term C is proportional to the change of temperature advection in the vertical, which is often called differential temperature advection.

Qualitatively,

$$\left(\frac{\text{geopotential}}{\text{tendency}}\right) \propto -\left(\frac{\text{absolute vorticity}}{\text{advection}}\right) + \left(\frac{\text{temperature}}{\text{advection}}\right)_{lower} - \left(\frac{\text{temperature}}{\text{advection}}\right)_{upper}.$$

The role of vorticity advection was discussed earlier, and need not be repeated except to note that geopotential change is proportional to vorticity change with the opposite sign. (Vorticity increase corresponds to geopotential decrease.)

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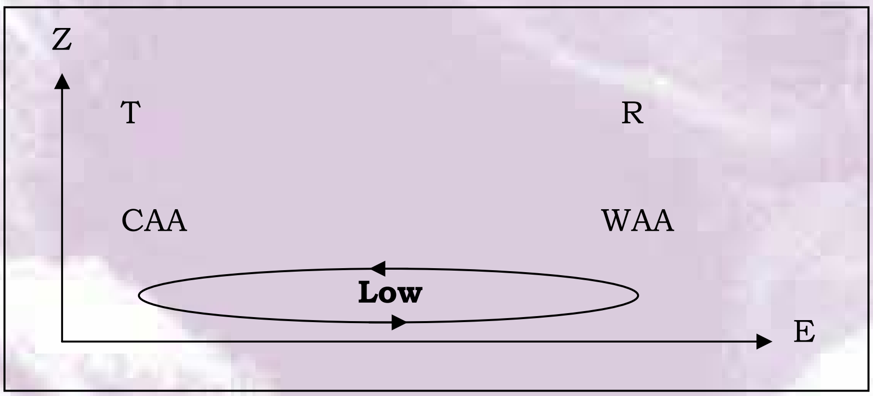
The temperature advection term is more complicated than whether the advection is warm or cold. How the temperature advection changes with altitude is important. Warm advection decreasing with height leads to rising geopotential, and cold advection decreasing with height leads to falling geopotential.

Usually, temperature advection is strongest slightly above the surface, between 850 and 700 mb, and approaches zero in the upper troposphere. The relative position of surface pressure centers to upper level troughs and ridges varies however.
Consider a surface low pressure center that is east of the mid-troposphere trough axis, and about half way between that trough and the downstream ridge. Under the ridge axis, surface winds are from the south and temperature advection is positive. Under the trough axis, surface winds are from the north and temperature advection is negative. According to the geopotential tendency equation, geopotential at the trough axis will fall while geopotential at the ridge axis will rise. Thus, the mid-troposphere trough and ridge will both intensify.



Similarly, if the surface low is west of the trough axis, warm advection will occur below the trough axis, geopotential will rise, and the trough will weaken. If the surface low is directly below either the trough or ridge axis, intensity should not change.

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