Mixing Length Concepts

The mixing length theory is often used to parameterize the vertical eddy flux of zonal momentum, which is one of several stresses,

$$\rho\,\overline{u'w'} = -\tau_{zx}.$$

The usual starting point is to assume that a stress is proportional to the vertical wind shear, as was done with molecular friction. Using the **eddy coefficient of viscosity** along the x axis, μ_{ex} , as the proportionality factor,

$$\rho\,\overline{u'w'} = -\mu_{ex}\frac{\partial \bar{u}}{\partial z}. \qquad (1)$$

In general, the eddy coefficient of viscosity is not constant. To find a suitable parameterization, consider a parcel at the level z. Its zonal momentum is assumed to be the same as that of the environment at that level, $\bar{u}(z)$. As the parcel rises to the level $z+\delta z$, it is assumed to conserve its zonal momentum. Then the eddy zonal momentum, which is the difference between the parcel momentum, $\bar{u}(z)$, and the environmental momentum, $\bar{u}(z+\delta z)$, is

$$u' = \bar{u}(z) - \bar{u}(z+\delta z).$$

Multiplying and dividing by the vertical displacement, δz, and using the familiar symbols for vertical wind shear, $\frac{\partial \bar{u}}{\partial z}$,

$$u' = \left[\frac{\bar{u}(z)-\bar{u}(z+\delta z)}{\delta z}\right]\delta z = -\frac{\partial \bar{u}}{\partial z}\,\delta z.$$

Next, a parameterization is developed for the eddy vertical momentum, w'. It should be similar to the zonal momentum, but must have the same sign as the vertical displacement,

$$w' = \left|\frac{\partial \bar{u}}{\partial z}\right|\delta z.$$

Substituting into equation (1),

$$-\mu_{ex}\frac{\partial \bar{u}}{\partial z} = \rho\,\overline{u'w'} = \rho\,\overline{\left[-\frac{\partial \bar{u}}{\partial z}\,\delta z\right]\left[\left|\frac{\partial \bar{u}}{\partial z}\right|\delta z\right]}.$$

Or,

$$-\mu_{ex}\frac{\partial \bar{u}}{\partial z} = -\rho \left|\frac{\partial \bar{u}}{\partial z}\right| \frac{\partial \bar{u}}{\partial z} \overline{(\delta z)^2}.$$

Solving for the eddy coefficient of viscosity,

$$\mu_{ex} = \rho \left|\frac{\partial \bar{u}}{\partial z}\right| \overline{(\delta z)^2}.$$

Finally, the **mixing length** in the x direction, l_x, is defined from,

$$l_x^2 = \overline{(\delta z)^2}.$$

Then,

$$\mu_{ex} = \rho\, l_x^2 \left|\frac{\partial \bar{u}}{\partial z}\right|.$$

And the parameterization of the vertical eddy flux of zonal momentum becomes,

$$\boxed{\rho\, \overline{u' w'} = -\tau_{zx} = -\mu_{ex}\frac{\partial \bar{u}}{\partial z} = -\rho\, l_x^2 \left|\frac{\partial \bar{u}}{\partial z}\right| \frac{\partial \bar{u}}{\partial z}}.$$

The vertical eddy flux of zonal momentum can now be computed from large scale quantities, except for the mixing length. In general, the mixing length depends on several quantities such as stability and distance from the ground. How these factors are resolved depends upon the nature of the phenomena under consideration.