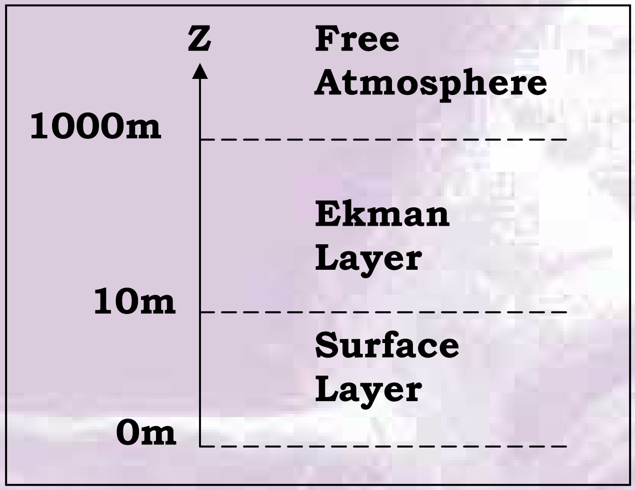
Ekman Layer

The layer of atmosphere from the top of the surface layer to the gradient wind level is called the **Ekman or Friction Layer**. Since the **gradient wind level** is where the effect of surface friction becomes relatively small, it is defined as the level where the wind direction first equals the direction of the geostrophic wind at the top of the PBL. The approximate altitudes of the bottom and top of this layer are 10 and 1000 meters, respectively.



The variation of wind with height in the Ekman Layer can be found using the equation of motion, with altitude as the vertical co-ordinate,

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + f v + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial z^2}$$

and

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - f u + \frac{\mu}{\rho}\frac{\partial^2 v}{\partial z^2}.$$

Assume a balance between the pressure gradient, Coriolis, and frictional forces,

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} + f v + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial z^2}$$

and

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} - f u + \frac{\mu}{\rho}\frac{\partial^2 v}{\partial z^2}.$$

Assume the pressure gradient and density are constant with height in the Ekman layer, which means that the geostrophic wind is also constant with height. Introduce the ageostrophic wind,

$$u = u_g + u_a \quad \text{and} \quad v = v_g + v_a,$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} + f\left(v_g + v_a\right) + \frac{\mu}{\rho}\frac{\partial^2 u_a}{\partial z^2}$$

and

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} - f\left(u_g + u_a\right) + \frac{\mu}{\rho}\frac{\partial^2 v_a}{\partial z^2}.$$

Or,

$$0 = +f v_a + \frac{\mu}{\rho}\frac{\partial^2 u_a}{\partial z^2}$$

and

$$0 = -f u_a + \frac{\mu}{\rho}\frac{\partial^2 v_a}{\partial z^2}.$$

Assuming that the eddy coefficient of viscosity does not change with height allows a new constant, a, to be defined,

$$2a^2 = \frac{f\rho}{\mu}.$$

The result is two equations with two unknowns, u_a and v_a, and one independent variable, z,

$$0 = +2a^2 v_a + \frac{\partial^2 u_a}{\partial z^2}$$

and

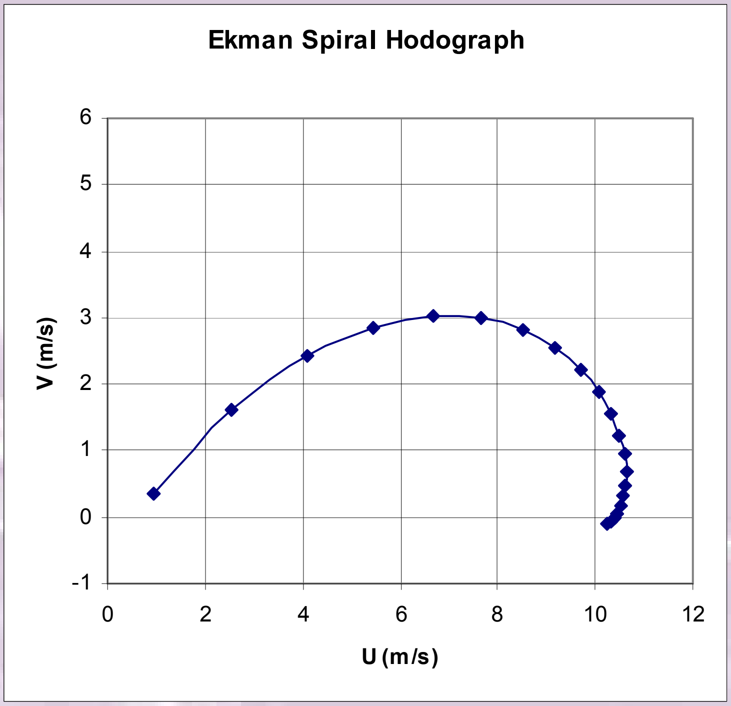
$$0 = -2a^2 u_a + \frac{\partial^2 v_a}{\partial z^2}.$$

At the lower boundary, z = 0, the ageostrophic wind components are, u_{ao} and v_{ao}. The upper boundary condition is that the wind approaches the geostrophic wind, or that the ageostrophic wind approaches zero, with increasing height.

When the x axis is rotated to the direction of the geostrophic wind, the solution is the **Ekman Spiral**.

$$u = u_g + e^{-az}\left[u_{ao}\cos az + v_{ao}\sin az\right]$$

$$v = e^{-az}\left[v_{ao}\cos az - u_{ao}\sin az\right].$$



A hodograph of a typical Ekman spiral is shown to the left. The geostrophic wind is 10 m/s; the surface wind is 1 m/s from 20 degrees counter clockwise of the geostrophic wind. Points are plotted for values of, az, from 0 to π radians, every π/18.

The wind speed at the surface is usually less than that in the free atmosphere, since friction tends to reduce the wind speed. The magnitude of the Coriolis force is therefore less than the pressure gradient force, and the surface wind direction will be slightly across the isobars, towards lower pressure. The angle between the surface wind and the isobars, the **cross isobar flow angle**, varies with the roughness of the surface. Over the ocean, 10 degrees is common, while over land the angle can be as large as 45 degrees.

As altitude increases, the wind direction veers with height and the wind speed increases. Near az = π/4, the cross isobaric wind speed reaches a maximum. Near az = π, the wind direction approaches that of the geostrophic wind.