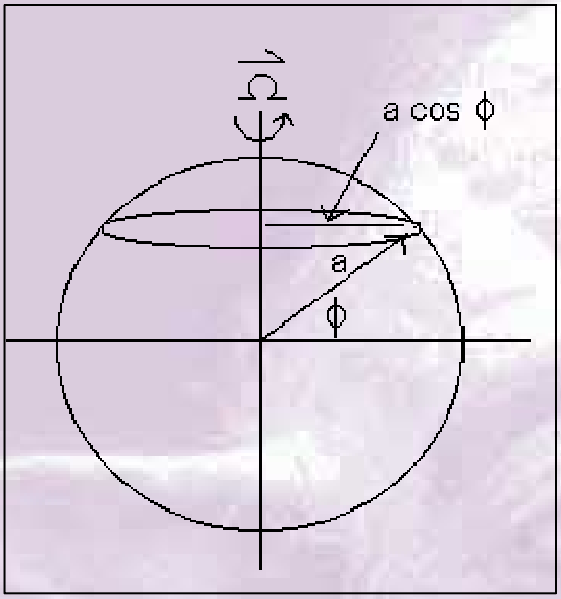
Conservation of Absolute Angular Momentum



For simple two-dimensional circular motion, the linear velocity, v, angular velocity, ω , and radius, r, are related by,

$$v = \omega r.$$

As the Earth rotates about its axis with angular velocity, Ω, a stationary parcel at latitude ϕ and altitude z, will travel in a circular path with radius, R,

$$R = (a + z)\cos\phi.$$

Within the troposphere, altitude is less than one percent of the Earth's radius, a, so,

$$R \approx a\cos\phi.$$

The velocity of the Earth's surface, u_e, is,

$$u_e = \Omega R = \Omega a\cos\phi.$$

The eastward speed of a parcel relative to the Earth's surface, u_r, is the eastward component of the wind, or **zonal wind**, u,

$$u_r = u.$$

The often used convention that the **absolute component equals the Earth's component plus the relative component**, can be applied to the zonal wind,

$$u_a = u_e + u_r = \Omega R + u = \Omega a\cos\phi + u.$$

The absolute angular momentum, m, is defined as,

$$m = R u_a = a\cos\phi\left(\Omega a\cos\phi + u\right). \quad (1)$$

m could also be written as the sum of the Earth's component and a relative component

$$m = m_e + m_r = \Omega R^2 + uR.$$

It can be proven that parcels conserve their absolute angular momentum,

$$\frac{dm}{dt} = 0;$$

or, as a parcel moves from position 1 to position 2,

$$m_1 = m_2 .$$

Using equation (1),

$$a\cos\phi_1\left(\Omega a\cos\phi_1 + u_1\right) = a\cos\phi_2\left(\Omega a\cos\phi_2 + u_2\right).$$

Or, solving for the final value of the zonal wind,

$$\boxed{u_2 = \frac{\cos\phi_1}{\cos\phi_2}\left(\Omega a\cos\phi_1 + u_1\right) - \Omega a\cos\phi_2}$$

If a parcel should move north, conservation of absolute angular momentum implies that the parcels zonal wind will increase.