Pressure Tendency Equation

To identify the processes that lead to surface pressure change, start with the hydrostatic equation,

$$\frac{\partial p}{\partial z} = -\rho g.$$

Multiply by dz and assume that pressure changes more with altitude than any other independent variable,

$$dp = -\rho g\, dz.$$

Integrate from the surface (pressure is p_o and height is 0), to the top of the atmosphere (pressure is 0 and height is ∞),

$$\int_{p_o}^{0} dp = -\int_{0}^{\infty} \rho g\, dz,$$

$$p_o = \int_{0}^{\infty} \rho\, g\, dz.$$

Next, take the partial derivative with respect to time,

$$\frac{\partial p_o}{\partial t} = g \int_{0}^{\infty} \frac{\partial \rho}{\partial t}\, dz.$$

Substitute for the partial of density with respect to time from the equation of continuity,

$$\frac{\partial p_o}{\partial t} = -g \int_{0}^{\infty} \left[\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} \right] dz.$$

Break the integral into two integrals, one for the horizontal and one for the vertical derivatives,

$$\frac{\partial p_o}{\partial t} = -g \int_{0}^{\infty} \left[\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} \right] dz - g \int_{0}^{\infty} \frac{\partial \rho w}{\partial z}\, dz.$$

Assume that ρw changes more with altitude than any other independent variable, and note that w is zero at the ground while ρ is zero at the top of the atmosphere,

$$\frac{\partial p_o}{\partial t} = -g\int_0^\infty \left[\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y}\right] dz - g\int_0^0 d\rho\, w.$$

The last integral is exactly zero because the limits of integration are zero.

Next, expand derivatives of products,

$$\frac{\partial p_o}{\partial t} = -g\int_0^\infty \left[u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + \rho\frac{\partial u}{\partial x} + \rho\frac{\partial v}{\partial y}\right] dz.$$

Or,

$$\frac{\partial p_o}{\partial t} = -g\int_0^\infty \left[u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y}\right] dz - g\int_0^\infty \left[\rho\frac{\partial u}{\partial x} + \rho\frac{\partial v}{\partial y}\right] dz.$$

To eliminate the partial derivatives of density, take the partial derivative of the equation of state,

$$\rho = \frac{p}{RT}.$$

$$\frac{\partial \rho}{\partial x} = \frac{\partial}{\partial x}\left(\frac{p}{RT}\right) = \frac{1}{R}\frac{\partial}{\partial x}\left(pT^{-1}\right) = \frac{1}{R}\left[T^{-1}\frac{\partial p}{\partial x} - pT^{-2}\frac{\partial T}{\partial x}\right].$$

So,

$$\frac{\partial \rho}{\partial x} = \frac{1}{RT}\frac{\partial p}{\partial x} - \frac{\rho}{T}\frac{\partial T}{\partial x} \quad \text{and} \quad \frac{\partial \rho}{\partial y} = \frac{1}{RT}\frac{\partial p}{\partial y} - \frac{\rho}{T}\frac{\partial T}{\partial y}.$$

Plugging in,

$$\frac{\partial p_o}{\partial t} = -g\int_0^\infty \left[u\left(\frac{1}{RT}\frac{\partial p}{\partial x} - \frac{\rho}{T}\frac{\partial T}{\partial x}\right) + v\left(\frac{1}{RT}\frac{\partial p}{\partial y} - \frac{\rho}{T}\frac{\partial T}{\partial y}\right)\right] dz$$

$$-\int_0^\infty \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right]\rho\, g\, dz$$

Rearranging, and switching vertical coordinates to pressure in two terms,

$$\frac{\partial p_o}{\partial t} = \int_{p_o}^{0} \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right] dp \quad + \int_{0}^{\infty} \left[\frac{g}{RT} \left(-u\frac{\partial p}{\partial x} - v\frac{\partial p}{\partial y} \right) \right] dz$$

$$+ \int_{p_o}^{0} \left(\frac{1}{T} \right) \left[-u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} \right] dp.$$

Thus,

$$\frac{\partial p_o}{\partial t} = \text{divergence effect + pressure advection effect}$$

$$\text{+ temperature advection effect.}$$

Surface pressure can be reduced by three physical processes: when a vertical column has a net horizontal positive divergence, negative pressure advection, or warm air advection.