Kinematic Method of Calculating Vertical Motion

One of the most widely used methods of calculating vertical motion is the kinematic method. Start with the flux form of the continuity equation,

$$\frac{\partial \rho}{\partial t} = -\left[\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z}\right].$$

Expand derivatives of a product and regroup terms.

$$\frac{\partial \rho}{\partial t} = -\left[\rho\frac{\partial u}{\partial x} + u\frac{\partial \rho}{\partial x} + \rho\frac{\partial v}{\partial y} + v\frac{\partial \rho}{\partial y} + \rho\frac{\partial w}{\partial z} + w\frac{\partial \rho}{\partial z}\right].$$

$$\frac{\partial \rho}{\partial t} = -\left[u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z}\right] - \rho\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right].$$

$$\frac{\partial \rho}{\partial t} + \left[u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z}\right] = -\rho\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right].$$

$$\frac{1}{\rho}\frac{d\rho}{dt} = -\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right]. \qquad (1)$$

Next, assume the atmosphere is **incompressible**, or the volume of a parcel is constant. Since a parcel has constant mass, the density is also constant. Thus,

$$\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right] = 0.$$

We want to solve for the vertical velocity , w ,

$$\frac{\partial w}{\partial z} = -\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right].$$

Multiply both sides by dz ; and assume w changes more with z than with any other independent variable.

$$dw = -\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right] dz$$

Thus, vertical velocity can be computed if the horizontal divergence is known. This is essentially the **kinematic method**.

If pressure is chosen as the vertical co-ordinate, equation (1) becomes (without any assumptions or approximations),

$$\frac{\partial \omega}{\partial p} = -\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right], \text{ and } \qquad \boxed{d\omega = -\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right] dp}$$