## Geostrophic Wind

The geostrophic wind is a hypothetical wind that results in a Coriolis force that exactly balances the horizontal pressure gradient force. It is a good approximation to the actual wind through most of the troposphere where friction is not significant.

To find the geostrophic wind components, $u_g$ and $v_g$, start with the equation of motion scaled for synoptic-scale circulations, with pressure as the vertical co-ordinate,

$$\frac{du}{dt} = -\frac{\partial \phi}{\partial x} + f v \text{ and } \frac{dv}{dt} = -\frac{\partial \phi}{\partial y} - f u. \qquad (1)$$

Assume that the acceleration is zero, and note that the wind that makes the Coriolis force balance the pressure gradient force is the geostrophic wind,

$$0 = -\frac{\partial \phi}{\partial x} + f v_g \text{ and } 0 = -\frac{\partial \phi}{\partial y} - f u_g. \qquad (2)$$

Rearrange,

$$f v_g = +\frac{\partial \phi}{\partial x} \text{ and } f u_g = -\frac{\partial \phi}{\partial y}. \qquad (3)$$

The geostrophic wind is especially useful if it has no divergence. Thus, the Coriolis parameter is assumed constant in equation (3),

$$\boxed{f_o v_g = +\frac{\partial \phi}{\partial x} \text{ and } f_o u_g = -\frac{\partial \phi}{\partial y}}$$

To find the divergence, take the partial derivative of each component as follows,

$$\frac{\partial}{\partial y}\left(f_o v_g\right) = +\frac{\partial}{\partial y}\frac{\partial \phi}{\partial x}$$

and

$$\frac{\partial}{\partial x}\left(f_o u_g\right) = -\frac{\partial}{\partial x}\frac{\partial \phi}{\partial y}.$$

Next, add these two equations together,

$$f_o\left(\frac{\partial u_g}{\partial x} + \frac{\partial v_g}{\partial y}\right) = -\frac{\partial}{\partial x}\frac{\partial \phi}{\partial y} + \frac{\partial}{\partial y}\frac{\partial \phi}{\partial x}.$$

And since the order of partial differentiation does not matter,

$$\boxed{f_o\left(\frac{\partial u_g}{\partial x} + \frac{\partial v_g}{\partial y}\right) = 0}$$

As promised, the geostrophic wind has no divergence.

## *Stream function of the Geostrophic Wind*

Since the geostrophic wind has no divergence, the stream function exists. The relationship between the stream function, $\psi$, and the geostrophic wind is given by,

$$u_g = -\frac{\partial \psi}{\partial y} \text{ and } v_g = \frac{\partial \psi}{\partial x},$$

so that the wind is parallel to the stream lines and proportional to the gradient of the stream function. Thus, the wind is stronger when the stream lines are closer together.

From the definition of the geostrophic wind,

$$f_o u_g = -\frac{\partial \phi}{\partial y} \text{ and, } f_o v_g = +\frac{\partial \phi}{\partial x}$$

it follows that,

$$\frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x}\left(\frac{gz}{f_o}\right) \text{ and } \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y}\left(\frac{gz}{f_o}\right).$$

And,

$$\psi = \frac{gz}{f_o}.$$

Thus, the stream function of the geostrophic wind on an isobaric surface is a constant times the height. And lines of constant height are lines of constant stream function, or stream lines. This is one of the reasons why height is isolined on an isobaric surface - to help visualize the wind flow.