Analytic Models of Atmospheric Fields

A common approach in dynamic meteorology is to assume that fields can be represented by analytic functions. When these functions are substituted into the governing equations, additional information can be learned about the atmosphere.

These analytic functions should meet two primary requirements. First, they should generate a realistic structure of the atmosphere. Second, they should be easy to use - relatively simple, and easy to differentiate and integrate. The latter is particularly important, since the governing equations contain many complex mathematical operations. All too often, these requirements contradict each other. Thus, a compromise between the two is used.

The geopotential height of an isobaric level (often called the geopotential), ϕ, is analytically modeled more than any other quantity. Perhaps this is because height contours on an isobaric surface clearly show the wind pattern and because many important quantities can be determined from the geopotential pattern.

To achieve a high degree of reality, the model should include the mean thermodynamic structure, the jet stream, and waves.

Mean Thermodynamic Structure

The simplest representation of the mean thermodynamic structure, is given by its long-term, mean value at each isobaric level,

$$\phi = \bar{\phi}(p). \qquad (1)$$

Such values may be determined by the U.S. Standard Atmosphere.

A more precise analytic relationship could be used, but the added complexity is usually not justified. For example, in a constant lapse rate atmosphere, the hypsometric equation can be integrated,

$$\phi = \frac{gT_o}{\gamma}\left[1 - \left(\frac{P}{P_o}\right)^{\frac{R_d\,\gamma}{g}}\right].$$

Jet Stream

The simplest representation of the jet stream is to use the geostrophic wind relationship, which relates the wind speed to the geopotential. If the wind direction is west to east, and the wind speed is constant,

$$f_o U_o = -\frac{\partial \phi}{\partial y}. \qquad (2)$$

Not surprisingly, the geopotential must decrease at a constant rate towards the north. The subscript, o, refers to a reference latitude, where $y = 0$.

To make use of this information, multiply both sides by dy , and integrate. Then equation (1) becomes,

$$\phi = \bar{\phi}(p) - f_o U_o y.$$

Since the jet stream was assumed to be constant, it does not vary with latitude. In some investigations, this does not affect the results, although this is clearly unrealistic.

If realism is important, then the jet stream should decrease both north and south of the reference latitude. To accomplish this, U_o should be multiplied be some function that has the desired north-south behavior. A function that does so and is easy to differentiate is the cosine. If L_J is twice the width of the jet,

$$f_o U_o \cos\left[\frac{2\pi y}{L_J}\right] = -\frac{\partial \phi}{\partial y}$$

would replace equation (2). Again, multiply both sides by dy , and integrate. Then equation (1) would become,

$$\phi = \bar{\phi}(p) - f_o U_o \left(\frac{L_J}{2\pi}\right) \sin\left[\frac{2\pi y}{L_J}\right].$$

Another improvement is for the jet stream to decrease both upward and downward from some pressure level, P_J . If the speed of the jet stream is zero at some level, P_Z , equation (1) would become,

$$\phi = \bar{\phi}(p) - f_o U_o \left(\frac{L_J}{2\pi}\right) \sin\left[\frac{2\pi y}{L_J}\right] \cos\left[\frac{2\pi (P - P_J)}{4(P_Z - P_J)}\right].$$

Waves

Atmospheric waves generally vary with each of the four independent variables x, y, p, and t. Before attempting to model such a complex circulation, lets first consider a simple wave that varies only with x. If the amplitude is ϕ', and the east-west wavelength is L_x, then the wave is given by

$$\phi' \sin\left[\frac{2\pi x}{L_x}\right].$$

If this wave also moves to the east with speed, c,

$$\phi' \sin\left[\frac{2\pi}{L_x}(x-ct)\right].$$

Adding north-south variation is straight forward. If the north-south wavelength is L_y,

$$\phi' \sin\left[\frac{2\pi}{L_x}(x-ct)\right] \cos\left[\frac{2\pi y}{L_y}\right].$$

Vertical variation of the wave could be exactly the same as with the jet stream. Wave amplitude is usually greatest at the level of the jet stream, and decreases toward the ground. Thus,

$$\phi' \sin\left[\frac{2\pi}{L_x}(x-ct)\right] \cos\left[\frac{2\pi y}{L_y}\right] \cos\left[\frac{2\pi (P-P_J)}{4(P_Z-P_J)}\right].$$

Adding the wave component of geopotential to the jet stream and mean thermodynamic components,

$$\phi = \bar{\phi}(p) + \cos\left[\frac{2\pi (P-P_J)}{4(P_Z-P_J)}\right] \times$$

$$\left\{\phi' \sin\left[\frac{2\pi}{L_x}(x-ct)\right] \cos\left[\frac{2\pi y}{L_y}\right] - f_o U_o \left(\frac{L_J}{2\pi}\right) \sin\left[\frac{2\pi y}{L_J}\right]\right\}$$