Ageostrophic Wind

By now, the geostrophic wind should be well understood. And it is time to investigate wind deviations from geostrophic - the **ageostrophic wind**. To better understand this concept, assume that the wind can be separated into two parts: a geostrophic wind and an ageostrophic wind,

$$u = u_g + u_a \text{ and } v = v_g + v_a. \tag{1}$$

Substitute equation (1) into the right hand side (RHS) of the large-scale equation of motion,

$$\frac{du}{dt} = -\frac{\partial \phi}{\partial x} + f\left(v_g + v_a\right) = -\frac{\partial \phi}{\partial x} + f v_g + f v_a$$

and

$$\frac{dv}{dt} = -\frac{\partial \phi}{\partial y} - f\left(u_g + u_a\right) = -\frac{\partial \phi}{\partial y} - f u_g - f u_a.$$

The first two terms on the RHS add to zero according to the definition of the geostrophic wind,

$$\frac{du}{dt} = +f v_a \text{ and } \frac{dv}{dt} = -f u_a. \tag{2}$$

Thus, acceleration results from only the ageostrophic part of the wind. Further, an ageostrophic wind from the south (positive v_a) produces an acceleration towards the east (positive $\frac{du}{dt}$) and an ageostrophic wind from the west (positive u_a) produces an acceleration towards the south (negative $\frac{dv}{dt}$). The acceleration is always directed 90 degrees to the right of the ageostrophic wind.

An important cause of acceleration (and the ageostrophic wind) is height (or pressure) change with time (tendency). To find such a relationship, first solve equation (2) for the ageostrophic wind,

$$u_a = -f^{-1}\frac{dv}{dt} \text{ and } v_a = +f^{-1}\frac{du}{dt}.$$

Next, expand the total derivative on the RHS using the chain rule,

$$u_a = -f^{-1}\frac{dv}{dt} = -f^{-1}\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \omega\frac{\partial v}{\partial p}\right)$$

and

$$v_a = + f^{-1} \frac{du}{dt} = + f^{-1} \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \omega \frac{\partial u}{\partial p} \right).$$

Substitute for u and v in the local tendency terms from equation (1),

$$u_a = - f^{-1} \left(\frac{\partial v_g}{\partial t} + \frac{\partial v_a}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \omega \frac{\partial v}{\partial p} \right) \text{ and}$$

$$v_a = + f^{-1} \left(\frac{\partial u_g}{\partial t} + \frac{\partial u_a}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \omega \frac{\partial u}{\partial p} \right).$$

Substitute for the geostrophic wind from its definition,

$$u_a = - f^{-1} \left(\frac{\partial}{\partial t} \left(f_o^{-1} \frac{\partial \phi}{\partial x} \right) + \frac{\partial v_a}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \omega \frac{\partial v}{\partial p} \right) \text{ and}$$

$$v_a = + f^{-1} \left(\frac{\partial}{\partial t} \left(- f_o^{-1} \frac{\partial \phi}{\partial y} \right) + \frac{\partial u_a}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \omega \frac{\partial u}{\partial p} \right).$$

Rearranging,

$$u_a = - \left(f^{-2} \frac{\partial}{\partial t} \left(\frac{\partial \phi}{\partial x} \right) + f^{-1} \left(\frac{\partial v_a}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \omega \frac{\partial v}{\partial p} \right) \right) \text{ and}$$

$$v_a = + \left(- f^{-2} \frac{\partial}{\partial t} \left(\frac{\partial \phi}{\partial y} \right) + f^{-1} \left(\frac{\partial u_a}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \omega \frac{\partial u}{\partial p} \right) \right).$$

Finally,

$$u_a = - f^{-2} \frac{\partial}{\partial x} \left(\frac{\partial \phi}{\partial t} \right) - f^{-1} \left(\frac{\partial v_a}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \omega \frac{\partial v}{\partial p} \right) \text{ and}$$

$$v_a = - f^{-2} \frac{\partial}{\partial y} \left(\frac{\partial \phi}{\partial t} \right) + f^{-1} \left(\frac{\partial u_a}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \omega \frac{\partial u}{\partial p} \right).$$

The first term on the RHS, the contribution of height tendency to the ageostrophic wind, is called the **isallobaric wind**. (The name comes from a similar derivation that uses height as the vertical co-ordinate, and results in a similar expression that contains the pressure tendency.)

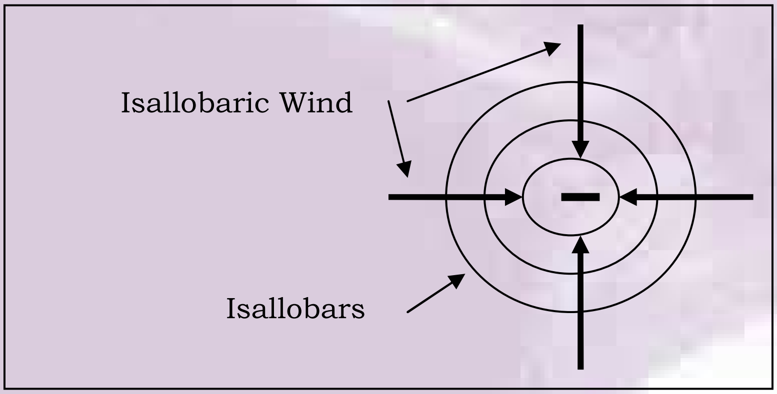
$$u_i = -f^{-2} \frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial t}\right) \text{ and } v_i = -f^{-2} \frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial t}\right),$$

or,

$$u_i = -\frac{1}{\rho f^2} \frac{\partial}{\partial x}\left(\frac{\partial p}{\partial t}\right) \text{ and } v_i = -\frac{1}{\rho f^2} \frac{\partial}{\partial y}\left(\frac{\partial p}{\partial t}\right) .$$

Accordingly, a horizontal gradient of pressure falls should lead to an ageostrophic component of the wind that is perpendicular to the isallobars and is directed towards the greatest pressure falls.

A particularly interesting meteorological situation is a circular region of pressure falls. Then the isallobaric wind should be directed towards the center of the pressure fall region from all directions, producing convergence.



To show that this is true, derive the divergence of the isallobaric wind. Assume that f and ρ are constant, and take the partial derivative of each wind component as shown below,

$$\frac{\partial}{\partial x}(u_i) = -\frac{1}{\rho f^2} \frac{\partial^2}{\partial x^2}\left(\frac{\partial p}{\partial t}\right)$$

and

$$\frac{\partial}{\partial y}(v_i) = -\frac{1}{\rho f^2} \frac{\partial^2}{\partial y^2}\left(\frac{\partial p}{\partial t}\right).$$

Then add the two equations together,

$$\frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y} = -\frac{1}{\rho f^2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial p}{\partial t}\right).$$

Thus, convergence should occur in a circular region of pressure falls (in addition to geostrophic rotation about low pressure). If this convergence is just above the ground, then the converging air must lead to ascent. When the pressure stops falling, the isallobaric wind, convergence, and ascent should cease as well.